PP36716. **Proposed by Pirkuliyev Rovsen**.

If $x \in \left(0, \frac{\pi}{4}\right)$, then prove:

$$\frac{4-\cos x}{\pi} < \frac{3\sqrt{2}}{4}.$$

Solution by Arkady Alt, **San Jose**,**California**, **USA**.

Since $\cos x \geq \frac{1}{\sqrt{2}}$ suffises to prove inequality $\frac{4-\frac{1}{\sqrt{2}}}{\pi} < \frac{3\sqrt{2}}{4} \Leftrightarrow$

$4-\frac{1}{\sqrt{2}} < \frac{3\pi}{2\sqrt{2}} \Leftrightarrow 8\sqrt{2}-2 < 3\pi \Leftrightarrow 8\sqrt{2} < 3\pi+2.$

Note that $3\pi+2 > 3 \cdot 3.14+2 = 11.42 > 11.4 = \frac{57}{5}$.

And since $8\sqrt{2} < \frac{57}{5} \Leftrightarrow 40\sqrt{2} < 57 \Leftrightarrow 3200 < 57^2 = 3249$ then

$8\sqrt{2} < \frac{57}{5} < 3\pi+2.$